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Faculty of Mathematics and Natural Sciences

Field of study: Mathematics

Financial risk modeling: Theoretical background and case-studies on market risk using historical simulations

Thesis submitted in fulfillment of the requirements for the Bachelor of Science degree in Mathematics (BSc.)

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Abstract

Determining the market risk using Value at Risk is currently generating plentiful of discussion in financial markets. The main objective of the first part of this thesis is to gain a good understanding of the basics of financial markets, valuation and risk modeling through literature study. In the second part of this work we consider case-studies related to quantification of market risk using an R implementation of historical simulations.

Acknowledgements

First, I would like to express my gratitude and appreciation to Prof. Dr. Drona Kandhai. His guidance, expertise and encouragement has given me the possibility to complete this study. I would also like to give my special thanks to my faculty professors and lecturers, especially, Mr. Gorisson and Dr. Venetiaan. Their continuous guidance has made it less difficult in deciding the area of Mathematics I wanted to graduate in. Finally, I would like to thank my family, friends and fellow students for all their support. Their understanding and encouragement has helped me a great amount in finishing my degree.

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Introduction

Value at risk methods are important and useful tools to quantify the market risk of portfolio's in financial markets. Being aware of the risk a portfolio poses, allows risk managers to help financial institutions to make better decisions. First, these risk managers have to detect which method is appropriate to apply in a certain portfolio. The first objective of this thesis was to gain a good understanding of the basics of financial markets, valuation and risk modeling through literature study. And, the second objective of this study was an R implementation of historical simulations on the market risk of three portfolio's.

The first chapter of this thesis describes some of the derivatives that appear in financial markets, such as options and forward contracts. In chapter 2, we continue with time series, with the main focus on Autoregressive time series models. Stochastic processes such as Wiener's process and Ito's lemma were beneficial to look at in Chapter 3, so the binomial trees and Black-Scholes model could be derived for valuation of options and portfolio's in chapter 4. Next, chapter 5 describes various Value at risk methods for estimating the Risk in a portfolio. Finally, we complete this study with the estimation of the VaR of foreign exchange rates using historical simulations.

Chapter 1

Derivative products

1.1 Forward contracts

In order to protect themselves two parties can agree ahead of time to transact at a specified price, regardless of what the market price of an asset is. This can be made official by setting up a contract where they agree to buy or sell an asset on a specified date for a certain specified price. This contract is a *forward contract*. For example, a coconut farmer produces half a million every year. This farmer may not be able to cover his costs when the demand for coconuts decreases, because that means that the price will drop. On the other hand we have a juices factory that specializes in making coconut juice and they cannot cover their costs when the prices of coconuts are too high. So, each party can experience a loss when the price of coconuts go below or higher than a certain price. Thus, in this case these two parties can decide to trade a forward contract.

In a forward 2 positions can occur:

- **long position** The party that agrees to buy the underlying asset assumes a long position.
- **short position** The party that agrees to sell the underlying asset assumes a short position.

Hedgers, speculators and sometimes arbitrageurs are the kind of buyers that engage in a forward contract. ***Hedgers*** are traders who protect themselves against risks that can occur in financial markets through derivatives, which are financial contracts whose value depend on the value of basic underlying assets. But a ***speculator*** trades derivatives with the prospect of making a profit. And a ***arbitrageur*** takes advantage of differences in price of the same thing in different markets to make risk-free profit.

1.1.1 Forward prices

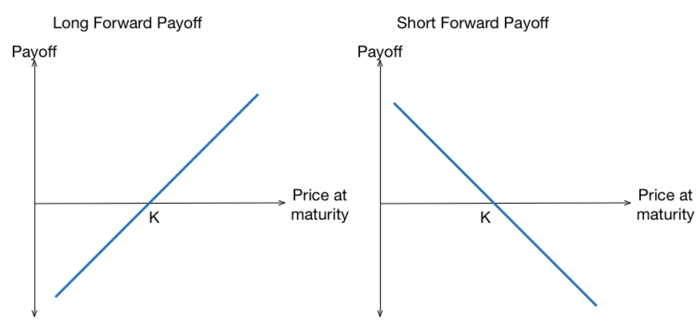
Generally, when you assume a long position in a forward contract on an asset, with delivery price K and spot price S_T, then payoff at expiry T is

$$S_T - K$$

The payoff when you assume a short position in a forward contract on an asset is

$$K - S_T$$

Figure 1.1: Payoffs from forward contracts



Example So consider that we, as arbitrageurs, want to buy a non-dividend-paying stock in 6 months on a long forward contract. Suppose that the stock price of one share is \$30 and the risk-free of interest rate (with continuous compounding) is 8% per annum. Then, the forward price is

$$30e^{0.08\times 6/12} = 31.22$$

In general, the price of a forward contract is

$$F_0 = S_0 e^{rT}$$

F_0 = forward price
S_0 = current spot price
r = risk-free rate per annum
T = delivery time in years

So, the forward price is in line with the spot price. This is because of the assumption that there is no arbitrage opportunity and no underlying dividend costs. Therefore, this not a riskless investment and the investor cannot resell the stock higher in a different market. If the forward is out of line with the spot price, then there exists a arbitrage opportunity. Thus, when the investor resells the stock they will make a riskless profit.

1.2 Put and Call options

There are two types of options in the over-the-counter market. A call option gives the holder the right to buy an underlying asset at a certain expiration date for a certain price. And a put option gives the holder the right to sell an underlying asset at a certain expiration date for a certain price. In both of these cases there is no obligation. The strike price or expiration price is the price in the contract. The difference between American options and European options is, that an American option can be exercised at any time up to the expiration date, while a European option can be exercised only on the expiration date itself.

1.2.1 Examples

The following examples give an explanation of how European put and call options work.

European call option An investor wants to buy 50 shares for \$8 through a European put exchange, meaning that the total cost is worth \$400. Currently, the price of the stock is \$52 and the strike price is \$50. As mentioned

before, we know that the investor can only exercise on the expiration date, because the option is European. Logically, the investor will only choose to trade if the stock price is more than \$50. The investor will make a profit if the price is more than \$58.
p = price on the expiration date to make a profit
p = \$50 + \$8 = \$58

Let's say the investor decides to trade at a price of \$65. So, the total cost of the 50 shares will be \$3250 (\$65 × 50 = \$3250). Then, the profit per share is \$15 and the total profit is \$750 (\$15 × 50 = \$750). Including the initial investment he will make a profit of \$350.
But when the price is \$55 on the expiration date, the investor will suffer a loss of \$150 (50 × \$5 - \$400 = \$250 - \$400 = - \$150)

European put option SMW (fictive investor) wants to sell 100 shares for \$6 through a European put exchange, meaning that the total cost is worth \$600. Currently, the price of the stock is \$75 and the strike price is \$80. Logically, the investor will only choose to trade if the stock price is less than \$80. The investor will make a profit if the price is less than \$74.
p = \$80 - \$6 = \$74

Let's say SMW still decides to trade at a price of \$60. The profit per share is \$20 and the total profit is \$2000 ((\$80 - \$60)× 100 = \$2000). Taking the initial cost into account, the profit will be \$1400. But when the price is \$85 on the expiration date, the investor will suffer a loss of \$1100 (\$80 - \$85 = - \$5 and (100 × -\$5) - \$600 = -\$1100). Thus, in this situation the investor will not exercise the option and only loses the premium.

Chapter 2

Time series

A time series is a sequence of measurements of data points in successive order. Time series analysis is useful for extracting necessary statistics and other characteristics of the data. A time series model will generally give a better prediction of observations in the near future. Often, time series models are used in Econometrics and Mathematical finance, Statistics, Modeling, Meteorology and Hydrology. The Autoregressive (AR) model is one of the most widely used time series models, therefore I shall discuss this later in this chapter. But firstly, we will look at the definition of time series along with the different types there are and some examples.

Definition 2.1 We can mathematically define time series as a set of random variables $\{X_t\}$, t = 0, 1, 2, ... where t represents the time period.

2.1 Types of times series models

Based on the type of variable a time series model contains, we can classify it in 2 two groups:

1. ***Univariate***
   When a model only consists observations of a single variable recorded over regular time intervals. i.e. weekly returns data of sock.

2. ***Multivariate***
   When a model has observations of more than one variable over the

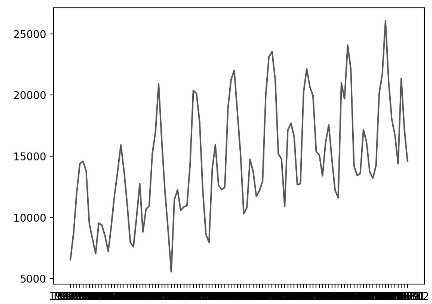


Figure 2.1: A time series graph, of monthly car sales of a random company, with a positive trend

same time period. i.e. The SBS collects data of 300 single moms on how much they spend on groceries in the past 3 months.

According to the rate of occurrences in the data (hourly, daily, weekly, monthly, annually, etc) different patterns may appear in time series graphs. The graphs can have a decreasing or increasing behavior over time with a constant slope or there may be patterns around the slope. These behaviors are described as the components: trend, seasonal and cyclic:

- **Trend** Continuous increase or decrease in data over a long time period, which is not always linear. When the values are increasing it is called a positive trend and a negative trend when the values are decreasing.

- **Seasonal** Predictable or regular fluctuations that occur over the course of a year, such as monthly or quarterly. Typically in a specific period of the year. i.e. in the months in the summer there is a large consumption of ice cream.

- **Cyclic** Wave fluctuations which occur for periods over longer than a year. These fluctuations are rarely regular and are usually a result of economic conditions.

- **Irregular** Fluctuations that cannot be explained by the trend, seasonal and cyclical movements. They can also be described as 'accidental' influences.

The following figures are different examples of the four components combined.

Figure 2.2: shows strong seasonality in the monthly housing sales, as well as some strong cyclic behavior

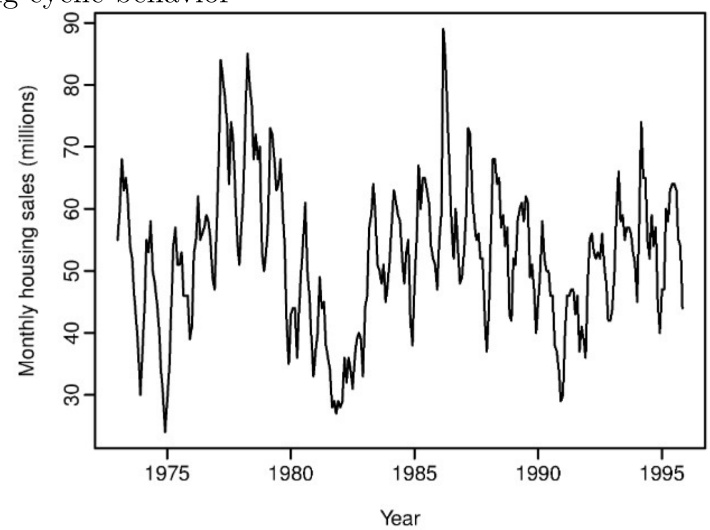


Figure 2.3: shows a downward trend in the US treasury bill contracts

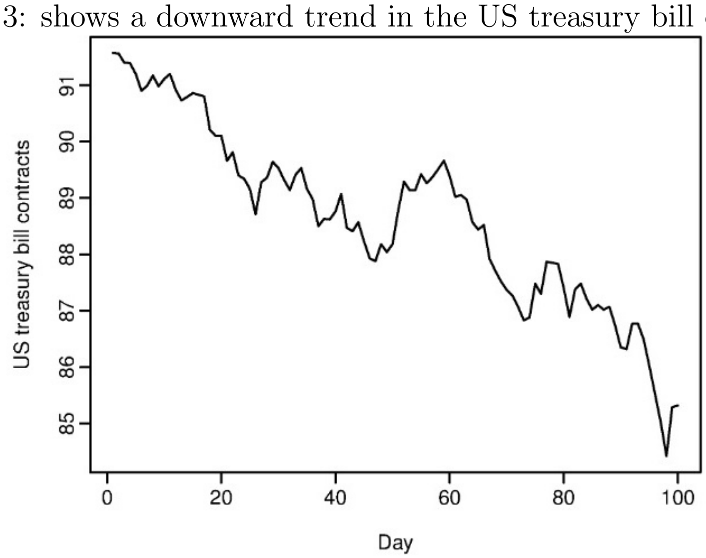


Figure 2.4: shows a strong increasing trend, with strong seasonality in the Australian quarterly electricity production

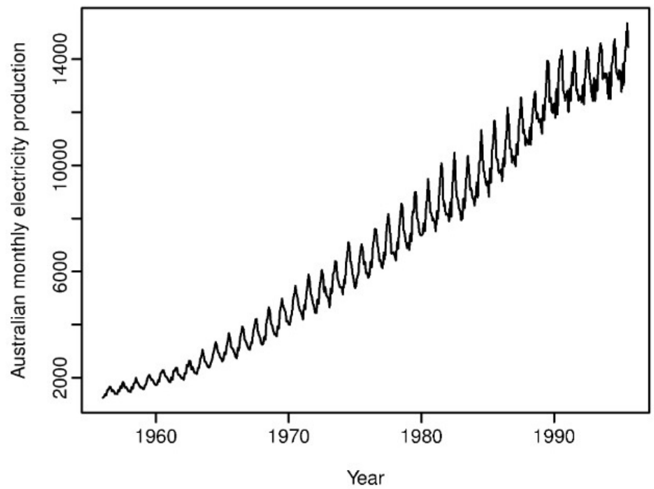
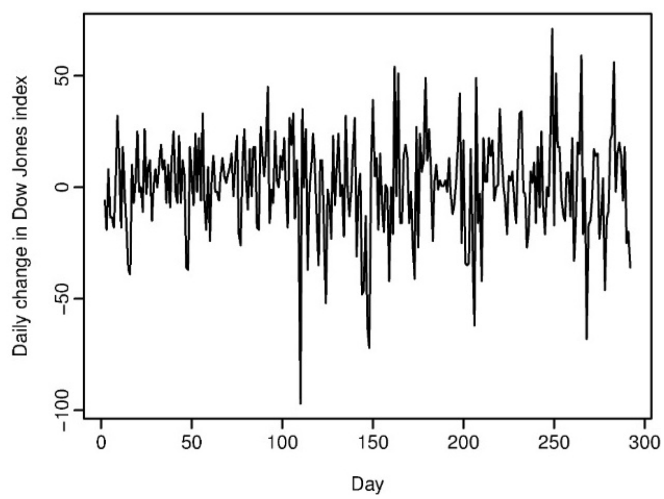


Figure 2.5: shows no trend, seasonality or cyclic in the Google closing stock price



If the current value of the series is a linear function of previous observation then the time series model is said to be linear and non-linear when the current value of the series is a non-linear function. In a continuous time series observations are measured at every instance of time, whereas a discrete time series contains observations measured at discrete points in time.

2.2 Local examples

In this section we have the time series graphs of USD-SRD exchange rates, crude oil and gold prices.

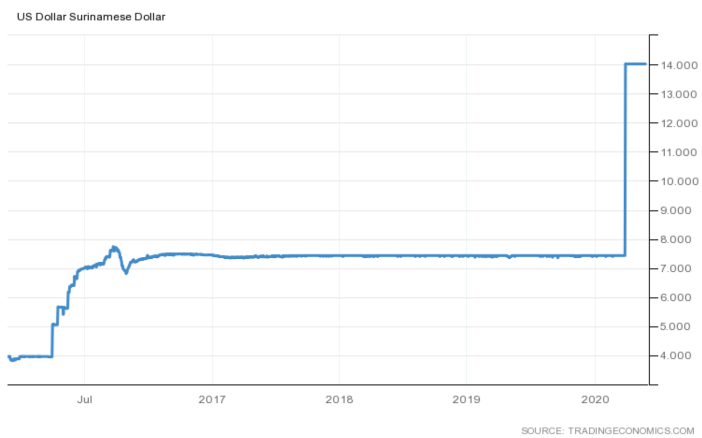


Figure 2.6: USD-SRD exchange rates from december 2015 - november 2020 (Source: Trading Economics)

In figure 2.6 there is no trend, seasonality or cyclic behavior. However, there is a huge increase of the exchange rates in 2015 and 2020, which are a result of inflation and economic recession.

Figure 2.7: USD Gold prices per oz from May 2016 - May 2021 (Source: Trading Economics)

Figure 2.7 shows a upwards or positive trend and some seasonality in the USD gold prices. The increase of these prices are significantly impacted by COVID-19 due to an increase of uncertainty in financial markets, which has led to a bigger demand for gold. There is also a certain relationship between gold and crude oil prices. This means that when there is an increase or decrease in gold prices, crude oil will behave in the same matter. In figure 1.8 an upwards trend can clearly be seen.

Figure 2.8: USD crude oil prices per oil barrel from 2001-2021(Source: Trading Economics)

In figure 2.8 not only an upwards trend but also some cyclic behavior can be seen in the crude oil prices.

2.3 Measurement functions

Definition 2.2 The **mean function** is defined as

$$\mu_t = \mu_x t = \mathbb{E}[X_t] = \int_{-\infty}^{\infty} x f_t(x)\, dx$$

provided it exists, where $\mathbb{E}$ denotes the usual expected value operator.

Definition 2.3 The **autovariance function** is defined as

$$\gamma(s,t) = \gamma_x(s,t) = cov(X_s, X_t) = \mathbb{E}[(X_s - \mu_s)(X_t - \mu_t)],$$

for all s and t. In this case we assume that the variance of X_t is finite. The autocovariance is useful for measuring the linear dependence between two observations on the same series at different times. – Very smooth series exhibit autocovariance functions that stay large even when the t and s are far apart, whereas choppy series tend to have autocovariance functions that are nearly zero for large separations.

Definition 2.4 The **autocorrelation function (ACF)** is defined as

$$\rho(s,t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\gamma(t,t)}}$$

With the Cauchy-Schwarz inequality

$$|\gamma(s,t)|^2 \leq \gamma(s,s)\gamma(t,t),$$

we can simply show that $-1 \leq \rho(s,t) \leq 1$.

With the ACF the linear predictably of the time series at time t, using only the value X_s, can be measured. If X_t can be perfectly predicted from X_s by a linear relation, then the ACF is ± 1.

2.4 White noise and stationary

Sometimes in time series forecasting, we can come across the term *white noise*. White noise series are time series that show no autocorrelation, meaning that the amount of correlation (time lag) between the values is very close to zero. Gaussian white noise is white noise with independent normal random variables.

Properties of white noise process:

1. *The mean, $\mathbb{E}[Yt]$, is constant.*
2. *The variance, Var[Yt], is constant.*
3. *The auto-covariance is zero.*

Forecasting time series data is easier when the series is stationary. We speak of *stationarity* when the joint probability of a series does not change over time, which means the mean, variance remain constant over time and the is no seasonality. If the series is not stationary, it can be transformed using mathematical methods.
Stationarity can be distinguished by **strictly stationarity** and **weakly stationarity**.

Strict stationarity
The distribution of a stochastic process doesn't change over time when the process is strictly stationary. Thus, the joint distribution depends only on the difference in time and not the location in time.

Mathematically, if X is a discrete stochastic process with distribution F then,

$$F_X(x_{t_{1+h}}, ..., x_{t_{n+h}}) = F_X(x_{t_1}, ..., x_{t_n})$$

Weak stationarity
A weak stationary process has a constant mean and variance and the variance between two time points, Y_t and Y_{t-s}, is constant. The coavariance only depends on s, which is the difference between these two time points and not the location. In other words, X_t, $t \in \mathbb{Z}$ is weakly stationary if:

1. $\mathbb{E}[X_t] = \mu$*, for all* $t \in \mathbb{Z}$*;*
2. $\mathbb{E}[{X_t}^2] < \infty$*, for all* $t \in \mathbb{Z}$*;*
3. $\gamma_x(s,t) = \gamma_x(s+h),(t+h)$*, for all* $t, s, h \in \mathbb{Z}$*;*

2.5 Autoregressive Models

A Autoregressive model (AR) is a commonly used linear time series model, where past observation are needed to forecast the current value. This model explicitly works on stationary time series.

Definition 2.5 An autoregressive model of order p, abbreviated AR(p), can be written as

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + ... + \varphi_p X_{t-p} + w_t$$

where X_t is stationary and $X_{t-1}, X_{t-2}, ..., X_{t-p}$ are the past series values. $\varphi_1, \varphi_2, ..., \varphi_p$ are the parameters of the model and w_t is a white noise term with $w_t \sim N(0, \sigma_w^2)$.

2.5.1 AR(1) process

An AR(0) process is the simplest AR process, which only consists of white noise and has no dependence between the terms.
An first-order autoregression process, AR(1), is written as:

$$X_t = \varphi_1 X_{t-1} + w_t \tag{2.1}$$

From (2.1), we can see that only the previous term and the noise term are essential for the output. If $\mid \varphi_1 \mid < 1$, then the process is stationary. If $\varphi_1 = 1$, then X_t is not stationary, as its variance depends on t, and is therefore infinite.

Suppose that $\mid \varphi_1 \mid < 1$, then

The **Mean** is

$$\mathbb{E}(X_t) = 0 \tag{2.2}$$

Proof

$$\begin{aligned} X_t &= \varphi_1 X_{t-1} + w_t \\ &= \varphi_1(\varphi_1 X_{t-2} + w_{t-1}) + w_t \\ &= \varphi 1^t \, X_0 + \sum_{j=0}^{t-1} \varphi_1^j \, w_{t-j} \end{aligned} \tag{2.3}$$

Now, $\mathbb{E}[X_t] = \varphi_1^t \; \mathbb{E}[X_0] + \sum_{j=0}^{t-1} \varphi_1^j \; \mathbb{E}[w_{t-j}]$

The expectation of w_{t-j} is zero, so now we have: $\mathbb{E}[X_t] = \varphi_1^t \; E[X_0]$. X_t is stationary which means that this equation can only be true if $\mathbb{E}[X_0] = 0$. This results in $\mathbb{E}[X_t] = 0$.

The **Variance** is

$$Var(X_t) = \frac{\sigma_w^2}{(1-\varphi_1^2)} \tag{2.4}$$

Proof
We can easily see that,
$\sigma(X_t) = \varphi_1^2\,\sigma(X_{t-1}) + \sigma(w_t) + 2\,\varphi_1\ Cov(X_{t-1}, w_t)$.
X_{t-1} and w_t are uncorrelated, which means that $Cov(X_{t-1}, w_t) = 0$.
So,

$$\sigma(Xt) = \varphi_1^2\,\sigma(X_{t-1}) + \sigma_w^2 \tag{2.5}$$

Recall that the the process is stationary, i.e.

$$\sigma(X_{t-1}) = \sigma(X_t) \tag{2.6}$$

From(2.5) and (2.6) we get,

$$\sigma(X_t) = \frac{\sigma_w^2}{(1-\varphi_1^2)}$$

The **Autcorrelation Function (ACF)** is

$$\rho_h = \varphi_1^h \tag{2.7}$$

Proof

We can deduce that:
$X_{t+h} = \varphi_1^h\ X_t + \sum\limits_{j=0}^{h-1} \varphi^j\ w_{(t+h)-j}$

$Cov(X_{t+h}, X_t) = Cov(\varphi_1^h\ X_t + \sum_{j=0}^{h-1} \varphi_1^j\ w_{(t+h)-j}, X_t)$

There is no covariance between X_t and the error terms, so we have

$$\begin{aligned} Cov(X_{t+h}, X_t) &= \varphi_1^h\ Cov(X_t, X_t) \\ &= \varphi 1^h\ \sigma(X_t) \\ &= \varphi_1^h\ \frac{\sigma^2 w}{1-\varphi_1^2} \end{aligned} \tag{2.8}$$

$$\gamma(h) = Cov(X_{t+h}, X_t) = \varphi_1^h \, \frac{\sigma^2 w}{(1 - \varphi_1^2)}$$

$$\gamma(0) = Cov(X_t, X_t) = \sigma(X_t) = \frac{\sigma^2 w}{(1 - \varphi_1^2)}$$

Examples AR(1) processes

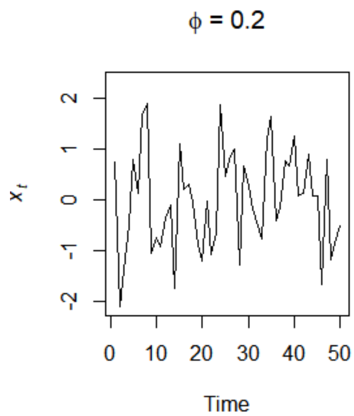


Figure 2.9: $X_t = 0.2X_{t-1} + w_t$

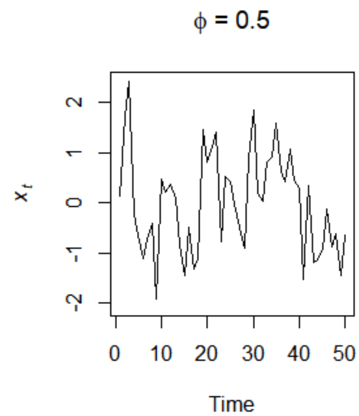


Figure 2.10: $X_t = 0.5X_{t-1} + w_t$

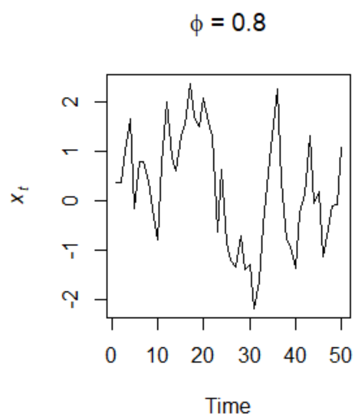


Figure 2.11: $X_t = 0.8X_{t-1} + w_t$

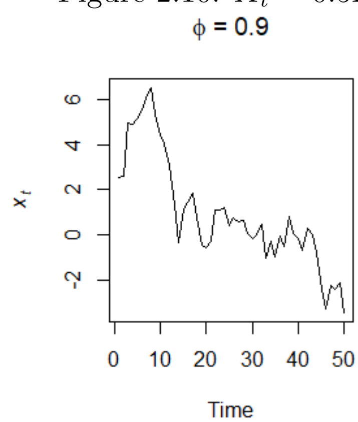


Figure 2.12: $X_t = 0.9X_{t-1} + w_t$

In these figures it is shown that when φ_1 is closer to 0 the more it will resemble white noise.
As there has been proven that $\rho_h = \varphi_1^h$, then figure 1.9 gives $\rho_h = (0.2)^h$.
And $\rho_h <<$, when $h >>$.

As we end this introduction on time series, it is also important to know that these series can represent stochastic processes. Chapter 3 follows with a discussion on this subject.

Chapter 3

Stochastic processes

Stochastic processes are often used in financial models for forecasting with random variables. A Markov process describes a sequence of possible outcomes (stochastic) where the future values only depend on the present value. Alluding that past values are irrelevant for this process. Usually, it is assumed that the changes in the prices of underlying assets (such as stocks) describe a Markov process. So, through this process we can determine the future price of an asset with the current price. In financial markets, this makes it a bit difficult for investors to trade without risk.
Geometric Brownian motion, a Wiener's process, and Itô's lemma are fundamental for the valuation of options, which is why I shall first consider the properties and results of these processes.

3.1 Wiener's process

A stochastic process, X_t, where X is a variable that changes over time and t the time can be distinguished by:

- A discrete variable with continuous time
- A discrete variable with discrete time
- A continuous variable with discrete time
- A continuous variable with continuous time

The Wiener process is a Markov stochastic process which has a continuous variable with continuous time. In physics this process is called *Brownian* *motion.*

The changes that the variable follows when it comes to a small interval Δt:

First property

$$\Delta z = \epsilon\sqrt{\Delta t} \tag{3.1}$$

ϵ is a standard normal variable with a mean of zero and a variance of 1.

Second property

The values of Δz for any two different periods of time are independent. From (3.1) it follows that Δz also has a standard normal distribution, meaning that:

- $E[\Delta z] = 0$
- Variance of $\Delta z = \sqrt{\Delta t}$
- Standard deviation of $\Delta z = \Delta t$

If $z(T) - z(0)$ is a Wiener process over a long period of time T, then

- $E[z(T) - z(0)] = 0$
- Variance of $[z(T) - z(0)] = T$
- Standard deviation of $[z(T) - z(0)] = \sqrt{T}$

When $\Delta \to 0$, $\sqrt{\Delta} \to 0$ at a much slower pace. Therefore, we will have changes in the Wiener process as Δt decreases to 0. The Wiener process stays contained when Δt is increased. Because of this we can observe a few properties:

1. The Wiener process is jagged at any proximity, so the path of this process is infinite.
2. Non-overlapping intervals are infinite

Definition 2.1 Generalized Wiener process A Wiener process, Δx, is a stochastic process with a variance rate of 1 over a time interval Δt and is given by

$$\Delta x = a\Delta t + b\epsilon\sqrt{\Delta t},$$

where $\epsilon \sim N(0,1)$, a is the drift rate and b is the variability. The **properties** are:

- $E[\Delta x] = a\Delta t$
- Variance of $\Delta x = b\sqrt{\Delta t}$
- Standard deviation of $\Delta z = b^2\sqrt{\Delta t}$

3.2 Geometric Brownian motion of a stock price

Brownian motion also known as the Wiener process was named after the botanist Robert Brown. This process was originally the motion of random particles inside gas or fluid and is necessary to derive the Black-Scholes model (Chapter 4).

$$\frac{\Delta S}{S} = \mu\Delta t + \sigma\varepsilon\sqrt{\Delta}t \tag{3.2}$$

Equation (2.2) is the formula of a discrete-time version of the Geometric Brownian motion model (figure 2.1), which can also be written as

$$\Delta S = \mu S\Delta t + \sigma S\varepsilon\sqrt{\Delta}t \tag{3.3}$$

ΔS = change in the stock price S
Δt = time interval
μ = expected rate of return from S
σ = volatility of S
$\mu\Delta t$ = expected rate of return from $\frac{\Delta S}{S}$
$\sigma\varepsilon\sqrt{\Delta}t$ = stochastic component of the return
$\sigma^2\Delta t$ = variance
$\sigma\sqrt{\Delta}t$ = standard deviation

If assumed that the changes in the stock price S have a normal distribution, the $\frac{\Delta S}{S}$ also has a normal distribution. So,

$$\frac{\Delta S}{S} \sim N(\mu \Delta t, \sigma^2 \Delta t)$$

This will be shown in the remainder of this chapter. For this, we will first introduce Itô's Lemma.

3.3 Itô's Lemma

Let S be the stock price at time t. The behavior in the stock price can be described as $dS = \mu S dt + \sigma S dz$ where $dz = \epsilon\sqrt{dt}$ is a Wiener process. This equation represents the stock price following an Itô process. The variable μ is the stocks expected rate of return the stock price and it is expected to increase with μS over time. The variable σ is the volatility of the stock price and σ^2 is referred to as its variance rate.
If dz is zero then integrating dS between 0 and T, shows that the stock price increases exponentially rate over time.

Now we consider G, which is a function of S and t, say G = G(S,t). Because G is a function of S the stochastic variable S, G will have a stochastic component.
The Taylor series for G = G(S,t) gives:

$dG = \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial S}dS + \frac{\partial^2 G}{\partial^2 S}(dS)^2$

$= \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial S}dS + \frac{\partial^2 G}{\partial^2 S}(\mu S dt + \sigma S dz)^2$

$= \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial S}dS + \frac{\partial^2 G}{\partial^2 S}(\mu^2 S^2 (dt)^2 + \mu\sigma S^2 dt dz + \sigma^2 S^2 (dz)^2)$

$= \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial S}dS + \frac{\partial^2 G}{\partial^2 S}(\mu^2 S^2 (dt)^2 + \mu\sigma S^2 \epsilon dt\sqrt{dt} + \sigma^2 S^2 \epsilon^2 dt)$

Taking into account the infinitesimal nature of dt so that any power higher than one vanishes:

$$dG = \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial S}dS + \frac{\partial^2 G}{\partial^2 S}\sigma^2 S^2 \epsilon^2 dt$$

Because $\mathbb{E}[\epsilon^2] = 1$, we get:

$$\begin{aligned} dG &= \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial S}dS + \frac{1}{2}\frac{\partial^2 G}{\partial^2 S}\sigma^2 S^2 dt \\ &= \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial S}(\mu S dt + \sigma S dz) + \frac{1}{2}\frac{\partial^2 G}{\partial^2 S}\sigma^2 S^2 dt \\ &= (\frac{\partial G}{\partial t} + \frac{\partial G}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 G}{\partial^2 S}\sigma^2 S^2)dt + \frac{\partial G}{\partial S}\sigma S dz \end{aligned} \tag{3.4}$$

Thus Itô's lemma reveals that if a stock price follows a Itô's process, a function consisting S and t follows the above equation.

For example, let G = ln(S), then
$\frac{\partial G}{\partial t} = 0$, $\frac{\partial G}{\partial S} = \frac{1}{S}$, $\frac{\partial^2 G}{\partial^2 S} = -\frac{1}{S^2}$
$dG = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dz$

G = ln(S) follows a Wiener process, since μ and σ are constant. The drift rate is $(\mu - \frac{1}{2}\sigma^2)$, σ is the volatility and μ^2 is the variance rate.
The change in ln(S) between time 0 and some future time T has a normal distribution, with mean $(\mu - \frac{1}{2}\sigma^2)$T variance σT.
So,

$$ln(S_T)–ln(S_0) = ln\frac{S_T}{S_0} \sim \phi[(\mu - \frac{1}{2}\sigma^2)T, \sigma^2 T]$$

or

$$ln(S_T) \sim \phi[(ln(S_0) + \mu - \frac{1}{2}\sigma^2)T, \sigma^2 T]$$

where

$ln(S_T)$ = the stock price at time T
$ln(S_0)$ = the initial stock price
$ln(S_0) + \mu - \frac{1}{2}\sigma^2)T$ = the mean of $ln(S_T)$
σT = the standard deviation of $ln(S_T)$

$ln(S_T)$ is lognomally distributed, which means that $S_T > 0$

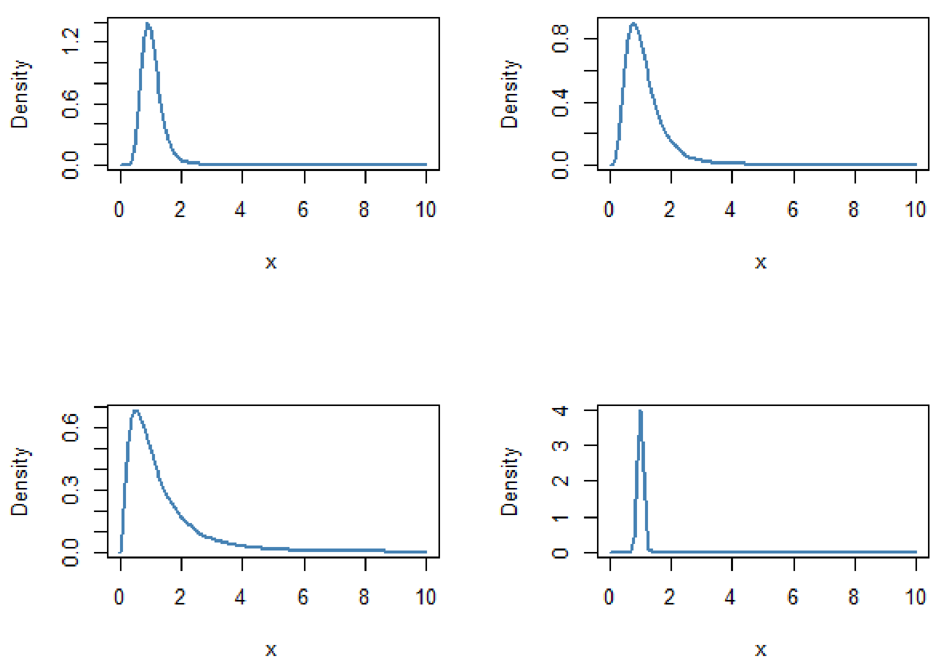
S_T is normally distributed because $ln(S_T)$ is normally distributed (according to the log-normal property).

3.4 Log-normal distribution

Log-normal distributions are defined as normal distributions of logarithmic values with parameters μ (mean) and σ (standard deviation). The definitions for variables that are log-normally distributed will be useful for derivation of the Black-Scholes model, that will be discussed later in another chapter. In

the figure below in each

Figure 3.1: **Log-normal density functions** are shown where the density gets higher as the standard deviation gets smaller.



Chapter 4

Binomial option pricing and the Black-Scholes model

The focus of this chapter will be on the pricing of options. The processes and results we have seen in the previous chapter are important keys for deriving the models that will be mentioned.

4.1 Pricing options with Binomial trees examples

The one-step binomial tree is one of the most useful models that is used to determine the value of a stock option. Later in this chapter, I will also discuss the Black-Scholes model which is another useful alternative when it comes to pricing options.
A binomial tree is a graphic representation of different possible values that the stock price may take at certain time periods.

By means of an example it will be explained how the approximation of the future price of an option can be done by making a construction of a binomial tree.

One-step binomial tree example The price of a equity (stock) is currently \$40. It is known that at the end of 1 month it will be either \$42 or \$38. The risk-free interest rate is 8% per annum with continuous compounding. We are interested in valuing a 1-month European call option with a strike

price of \$39. If the stock price is \$40 at the end of that one month, then the value of the option will be \$3. If it the stock turns out to be \$38, the value of the option will be \$0. I consider that arbitrage opportunities do not exist.
The delta (Δ) of a stock option is the ratio of the change in the price of the

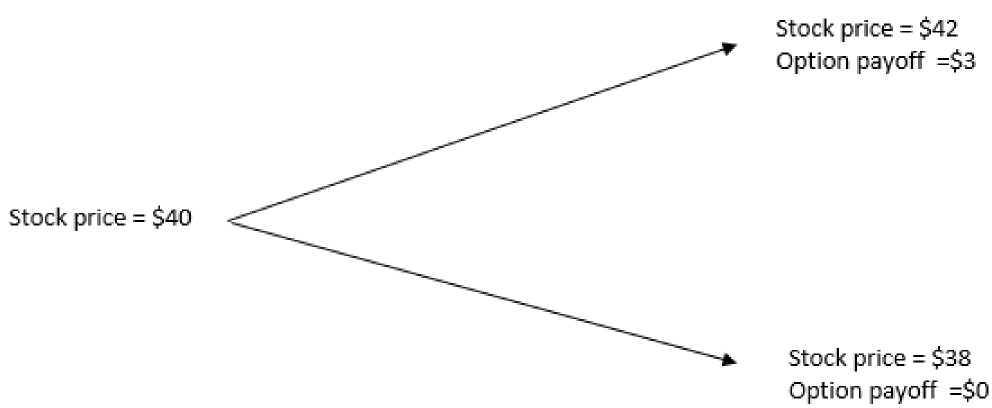


Figure 4.1: **Example One-step binomial tree**

stock option to change the price of the underlying shares. It is the number of units of the stocks we need to hold for each shorted option to create a risk-free portfolio. It is important to know at what value of delta the portfolio is without risk. The value of the shares is 42Δ and the option is the valued 3, when the price goes from \$40 to \$42. This means that the total value of the portfolio is 42Δ - 3. When the price reduces from \$40 to \$38, the value of the shares is 38Δ and the option has no value (value is zero). Then the total value of the portfolio is 38Δ. Trading the option is always riskless if Δ is chosen such that the end value does not change in both cases.
Calculating Δ by: $42\Delta - 3 = 38\Delta \Leftrightarrow \Delta = 0.75$

If the stock price increases to \$42, the value of the portfolio is

$$42 \times 0.75 - 3 = 28.5$$

If the stock price reduces to \$38, the value of the portfolio is

$$38 \times 0.75 = 28.5$$

The value of the portfolio is always 28.5 at the end of the life of the option, whether the stock price moves up or down. In this example the risk-free rate is 8% per annum. The present value of the portfolio is

$$28.5e^{-0.08\times 1/12} = 28.31$$

Currently the portfolio is valued at

$$f = 30 - 28.31 = 1.69$$

f = option price

4.2 One- Step generalization Binomial trees

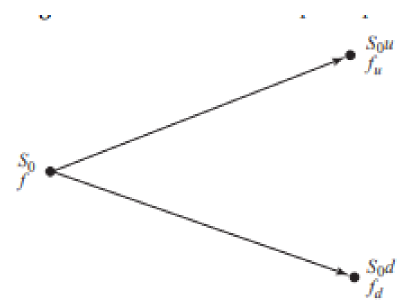


Figure 4.2: General one-step binomial tree

Consider a stock whose price is initially S_0. We want to derive the current price f of a European call option on the stock. Suppose that the option lasts for time T and that during the life of the option the stock price can either move up from S_0 to a new level, S_0u, or down from S_0 to a new level S_0d, where $u > 1$ and $d < 1$.
The payoff from the option is f_u if the stock price moves up and f_d if the stock price moves down.

We consider a portfolio consisting of a long position in Δ shares and a short position in one option. We calculate the value of Δ that makes the portfolio risk-free:

$$S_0u\Delta - f_u = S_0d\Delta - f_d \tag{4.1}$$

From we (4.1) we get:

$$\Delta = \frac{f_u - f_d}{S_0(u-d)} \tag{4.2}$$

The equation above shows that Δ is the ratio of the change in the option price to the change in the stock price as we move between nodes. The absence of arbitrage opportunities implies that the fair price of any investment is given by the present value of its future payoff. So we should have that the cost of setting up the above portfolio is equal to the present value of its future value. Since the portfolio has no risk, we should use the risk-free interest rate (r) to discount any future payments. In other words, a risk-free portfolio must earn the risk-free interest rate. Therefore, we have

$S_0\Delta - f = (S_0u\Delta - f_u)e^{-r\Delta t}$
$f = S_0\Delta - (S_0u\Delta - f_u)e^{-r\Delta t}$
$f = S_0\Delta(1 - ue^{-r\Delta t}) + f_ue^{-r\Delta t}$

Substitute Δ into the above to get:

$f = S_0\frac{f_u - f_d}{(S_0(u-d)}(1 - ue^{-r\Delta t}) + f_ue^{-r\Delta t}$

$f = \frac{(f_u - f_d)(1-ue-r\Delta t)+fue^{-r\Delta t}(u-d)}{(u-d)}$

$f = \frac{f_u(1-de^{-r\Delta t})+f_d(ue^{-r\Delta t}-1}{(u-d)}$

So,

$$f = e^{-r\Delta t}[pf_u + (1-p)f_d] \tag{4.3}$$

where $p = \frac{e^{r\Delta t}-d}{u-d}$.

Thus, (4.3) states that the value of the option today is its expected future value discounted at the risk-free interest rate. The expected stock price at time T is $E(S_T) = pS_0u + (1-p)S_0d = pS0(u-d) + S_0d$
$E(S_T) = (\frac{(e^{r\Delta t}-d)}{u-d})\,S_0(u-d) + S_0d = S_0e^{r\Delta t}$
The above shows that the stock price grows, on average, at the risk-free rate. Therefore, setting the probability of an up movement equal to p, is equivalent to assuming that the per annum rate of return on the stock equals the risk-free rate.

4.3 Black-Scholes model

In financial markets investors always want to know how much the stock price relatively differs to the strike price. The time they have to exercise the option with a certain risk-free interest and the volatility of the underlying stock are also important factors to know. Investors would most likely want to trade options that are more volatile, because those are more valuable.
In this section we are going to discuss the Black-Scholes model, a mathematical model, used in financial markets.

4.3.1 Black-Scholes-Merton formulas

The Black-Scholes-Merton formulas that can be deduced from the Black-Scholes differential equation are:

$$c = S_0 N(d2) - Ke^{-rt} N(d1) \tag{4.4}$$

$$p = Ke^{-rt} N(-d1) - S_0 N(-d2) \tag{4.5}$$

where c is the value of a European call option and p the value of a European put option.

And

$$d_1 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_2 - \sigma\sqrt{T}$$

and

$$d_2 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

Equation (4.4) can also be used to value an American call option, but a analytic formula to value an American put option on a non-dividend paying stock has yet to be produced. With the Black-Scholes model it can also be shown that the price of a forward contract is $S_0 e^{rT}$, by taking the difference of an put and call option.

Next, there are some probability terms we need mention before deriving the formulas.

Definition 4.1 The **Cumulative Distribution** of a random variable X for all real numbers b is defined by

$$F(b) = \mathbf{P}\{X \leq b\} = \int_{-\infty}^{b} f(x)\, dx$$

where f is the *probability density function* of X.

Definition 4.2 The **Cumulative Distribution Function** of a standard normal random variable X is given by

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-y^2}{2}}\, dy$$

and the density of X is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad for \quad -\infty < x < \infty$$

Definition 4.3 If X is a *continuous random random variable* with probability density function $f(x)$ then the **expectation** of X is

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

According to Ito's lemma the smooth function $G(S,t) = log(S_t)$ follows

$$d\, log(S_t) = (r - \frac{1}{2}\sigma^2)dt + \sigma dz_t \tag{4.6}$$

Here we have used the risk-neutral valuation where the expected return from the underlying asset μ is the risk-free interest rate r. Now integrating both sides we get

$$\int_t^T d\, log(S_u) = \int_t^T (r - \frac{1}{2}\sigma^2)du + \int_t^T \sigma dz_u \tag{4.7}$$

$$log(S_T) - log(S_t) = (r - \frac{1}{2}\sigma^2)(T-t) + \sigma(z_T - z_t) \tag{4.8}$$

By one of the properties of Brownian motion $z_T - z_t$ has a normal distribution with mean 0 and variance $T - t$. So if we indicate Z as standard normal variable, $z_T - z_t \simeq \sqrt{T-t}Z$.
Taking the exponential of equation 4.16 above we get

$$\frac{S_T}{S_t} = exp\,(r - \frac{1}{2}\sigma^2)(T-t) + \sigma\sqrt{T-t}Z$$

Now we can conclude that $\frac{S_T}{S_t}$ has a log-normal distribution.

The expectation of a European option at maturity using risk-neutral valuation is

$$[max(S_T - K, 0]$$

The expected value of the call option c discounted with the risk-free interest rate, because of risk-neutral valuation is

$$c = e^{-r(T-t)}\,\mathbb{E}[(S_T - K)^+] \tag{4.9}$$

Computing the expectation

First, let $\tau = T - t$ and for $-Z$ we'll write instead Z which is symmetrically equivalent. Then we substitute S_T in (A.4):

$$\begin{aligned}
c &= e^{-r\tau}\,\mathbb{E}[(S_T - K)^+] \\
&= e^{-r\tau}\,\mathbb{E}[(S_t\,exp\{(r - \frac{1}{2}\sigma^2)\tau - \sigma\sqrt{\tau}Z\} - K)^+] \\
&= \frac{e^{-r\tau}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(S_t\,exp\{(r - \frac{1}{2}\sigma^2)\tau - \sigma\sqrt{\tau}x\} - K)^+\,e^{-\frac{1}{2}x^2}\,dx \\
&= \frac{e^{-r\tau}}{\sqrt{2\pi}}\int_{-\infty}^{d1}(S_t\,exp\{(r - \frac{1}{2}\sigma^2)\tau - \sigma\sqrt{\tau}x\} - K)\,e^{-\frac{1}{2}x^2}\,dx \qquad (x < d1) \\
&= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d1}e^{-r\tau}\,S_t\,exp\{(r - \frac{1}{2}\sigma^2)\tau - \sigma\sqrt{\tau}x\}e^{-\frac{1}{2}x^2} - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d1}e^{-r\tau}K\,e^{-\frac{1}{2}x^2}\,dx
\end{aligned}$$

According to **Definition 4.2** it follows that

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d1}e^{-r\tau}K\,e^{-\frac{1}{2}x^2}\,dx = e^{-r\tau}KN(d1)$$

Further solving of the integral on the left and defining $y = x + \sigma\sqrt{\tau}$ gives

$$\frac{S_t}{\sqrt{2\pi}} \int_{-\infty}^{d1} exp\{-\frac{1}{2}(x^2 - \sigma^2\tau) - \sigma\sqrt{\tau}x\}\, dx$$

$$= \frac{S_t}{\sqrt{2\pi}} \int_{-\infty}^{d1} exp\{-\frac{1}{2}(x + \sigma\sqrt{\tau})^2\}\, dx$$

$$= \frac{S_t}{\sqrt{2\pi}} \int_{-\infty}^{d1+\sigma\sqrt{\tau}} exp\{-\frac{1}{2}y^2\}\, dy$$

$$= S_t N(d2)$$

where $d2 = d1 + \sigma\sqrt{\tau}$.

Chapter 5

Value at risk

In financial markets more often investors and financial managers are interested in financial risk beside the value of a portfolio. This chapter focuses on determining the Value at Risk (VaR), which is the total risk of a portfolio. There are 2 methods going to be discussed for estimating VaR, which are: Historical simulation and model-building approach.

The VaR tells a portfolio holder how much they are expected to lose at maximum over the next N days within a certain confidence level or probability.
Suppose that a portfolio holder wants to know what the maximum loss will be in 10 days with 95% certainty and the value determined is $100,000.
This is interpreted as: the holder is 95% confident that the loss in 10 days will be $100,000 or less. We can also say that there is a 5% chance that the minimum loss will be $100,000 or more. In this case 100,000 is the VaR (V) of the portfolio.
VaR has two parameters: N (days) and X (confidence level). An important assumption, when the returns of a portfolio are normally distributed, is:

$$N\!-\!day\ VaR = 1\!-\!day \times \sqrt{N}$$

5.1 Model-building approach

This approach assumes the probability distributions of the returns of the market variables. Actual data is used to estimate the model. When is using this approach the portfolio can consist of one stock or two stocks.

Some assumptions are that:

1. the expected change in the value of the portfolio is zero.
2. the change in the value of the portfolio has a normal distribution.

Single stock An investor has \$5 million in Google shares. He wants to know what the maximum loss will be over 20 days with 95% certainty, so N = 20 and X = 95. The standard deviation of the change in the portfolio in 1 day is $\$100,000$. This means that the standard deviation in 20 days is: $100{,}000\sqrt{20} = \$447{,}213$. $N^{-1}(0.05) = -1.645$, so the VaR for 1-day is

$$1.645 \times \$100{,}000 = \$164{,}500$$

And the 20-day 95% VaR is

$$\$164{,}500 \times \sqrt{20} = \$735{,}666$$

Two stocks Now 2 investors are interested in knowing the 95% VaR of the portfolio over 20 days. The portfolio consist of \$2 million and \$5 million shares from the investors. Suppose that the correlation between the returns is 0.2. From statistics it is known that the standard deviation of $X + Y$ is

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

if X and Y have standard deviations σ_X and σ_Y with ρ as the correlation between both.
Suppose that the day-to-day volatility of investor X is 1% and investor Y 2%, so

$$\sigma_X = 20,000 \; and \; \sigma_Y = 100,000$$

Then

$$\sigma_{X+Y} = \sqrt{20,000^2 + 100,000^2 + 2 \times 0.2 \times 20,000 \times 100,000} = 102,020$$

The VaR for 1-day is

$$1.645 \times \$102,020 = \$167,823$$

And the 20-day 95% VaR is

$$\$167,823 \times \sqrt{20} = \$750,527$$

The change in value of a portfolio has a linear relation with the returns of the market variables (normally distributed) and is given by

$$\Delta P = \sum_{i=1}^{n} \alpha_i \Delta x_i$$

with variance

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \alpha_i \alpha_j \sigma_i \sigma_j$$

or

$$\sigma_P^2 = \sum_{i=1}^{n} \alpha_i^2 \sigma_i^2 + 2 \sum_{i<j} \rho_{ij} \alpha_i \alpha_j \sigma_i \sigma_j$$

Δx_i = the return on asset i in one day
$\alpha_i \Delta x_i$= the invesment in asset i in one dayy
σ_i= the standard deviation of asset i
σ_p= the standard deviation of ΔP
σ_i^2= the variance of ΔP
α_i= the amount invested in asset i
α_j= the amount invested in asset j
ρ_{ij}= correlation coefficient between returns asset i and asset j

The variance of the portfolio return in one day, $\frac{\Delta P}{P}$, is

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \omega_i \omega_j \sigma_i \sigma_j$$

where ω_i is the weight in the portfolio of investment i.

5.2 Monte Carlo stimulation

The Monte Carlo simulation can be useful when applying the model-building approach for calculating the VaR. This method simulates the probability distribution of the change in value of the portfolio over a specific period of time.

After determining the value of the portfolio on the current day, a sample is taken from the returns on the ith asset. The sampled Δx_i's are then used to calculate the market variables at the end of that day to determine the new value of the portfolio. A sample ΔP is the difference between the old and the new of the portfolio on t day. And by repeating this process the probability distribution of ΔP can be created. Now, the 1-day VaR determined for N samples is defined as the value of ΔP for the $((100 - X)\% \times N)$th worst case.

5.3 Historical simulation approach

Historical simulation is the simplest approach of evaluating VaR. It uses historical data of any time in the past to predict what will occur in the future.
Before we calculate the VaR of a portfolio we need to identify important factors that can affect the portfolio such as the amount of money invested, interest rates etc.
If, for example, data is collected over 201 days then there are 200 different scenarios that can happen between Day i and Day (i+1). In scenario (i+1) the percentage changes of the variables are equal between Day i and Day (i+1).

$$ith\ scenario\ market\ value = v_n \frac{v_i}{v_{i-1}}$$

where v_i is the value of the market variables on Day i and n is the base date.

Chapter 6

Estimation of the Value at Risk of Foreign Exchange rates

In this section the Value at Risk will be estimated for EURO/SRD and EURO/USD exchange rates by using historical simulation. This approach is useful for financial institutions and even corporates such as gas and oil firms to estimate the daily market risk.

6.1 Methodology

First we created 3 hypothetical portfolio's, based on the currencies and periods. Let's say the first portfolio is worth 1 million EURO that is invested in USD exchange business. The EURO/USD exchange rates data used for this portfolio is from January 2, 2015, to October 15, 2015. The second portfolio is the same as the first portfolio but here we use data from April 1, 2020, to September 30, 2021. The historical data used in these first two portfolio's have been collected from Marcotrends research financial data. Now, the third portfolio is worth SRD 1 million that is invested in EURO exchange transactions. For this portfolio, historical data of EURO/SRD exchange rates from January 4, 2021, to September 1, 2021, has been collected from the Centrale Bank van Suriname.

Before the daily returns were computed, the data had been cleaned so it would not produce NA values. The first portfolio uses 201 days, whereas the second and third use data from respectively 469 and 156 days.

Next, the scenarios were created, the losses/returns and the value of the portfolios were computed. Through a histogram and normal Q-Q plot we identify the distribution of of the returns. And lastly, the estimation of the 1-Day 95% and 99% VaR of the portfolio's. For portfolio 1, 2 and 3 the 95% VaR estimation is the 10th, 23rd and 8th worst loss, respectively. And the 99% VaR for portfolio 1, 2 and 3 is estimated as the 2nd, 5th and 2nd worst loss, respectively.

6.2 Results and findings

6.2.1 Portfolio 1

Figure 6.1 shows that the biggest loss of this portfolio is 21,104. This loss falls under scenario 13. The histogram of returns, in figure 6.2, suggests that the daily returns of the EURO/USD exchange rates in the earlier mentioned before period, may be normally distributed. This plot only shows moderately skewness in the distribution. Our normal Q-Q plot, further suggests that the returns follow a normal distribution.

The VaR of Portfolio 1, in figure 6.4, is estimated:

- With 95% confidence we can say that the maximum 1-Day loss will be 14,204.79 €.
- With 99% confidence we can say the maximum 1-Day loss will be 19,781.03 €.

Figure 6.1: Summary of the returns and Portfolio 1 values

```
   nscenario         EURO.USD        portfolio           returns
 Min.   :  1.00   Min.   :1.115   Min.   : 978896   Min.   :-25958.1
 1st Qu.: 50.75   1st Qu.:1.133   1st Qu.: 994844   1st Qu.: -4244.1
 Median :100.50   Median :1.139   Median :1000000   Median :     0.0
 Mean   :100.50   Mean   :1.138   Mean   : 999768   Mean   :   232.2
 3rd Qu.:150.25   3rd Qu.:1.143   3rd Qu.:1004244   3rd Qu.:  5156.4
 Max.   :200.00   Max.   :1.168   Max.   :1025958   Max.   : 21104.3
```

Figure 6.2: Histogram of the returns (Portfolio 1)

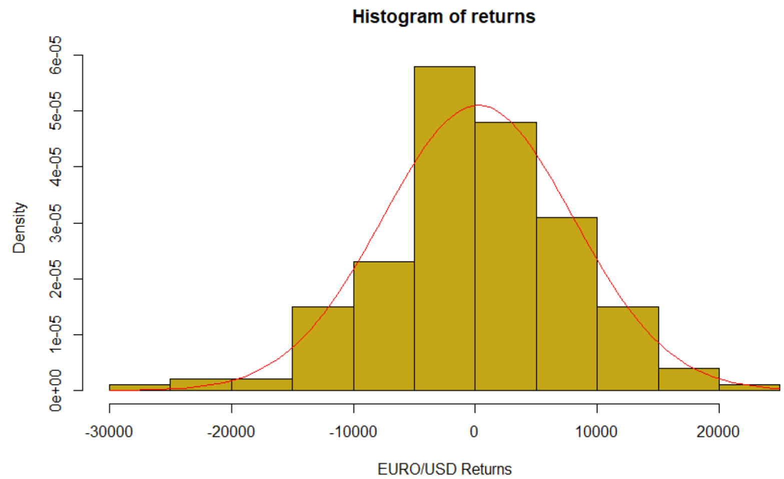


Figure 6.3: Q-Q plot of the EURO/USD exchange rates returns from Portfolio 1

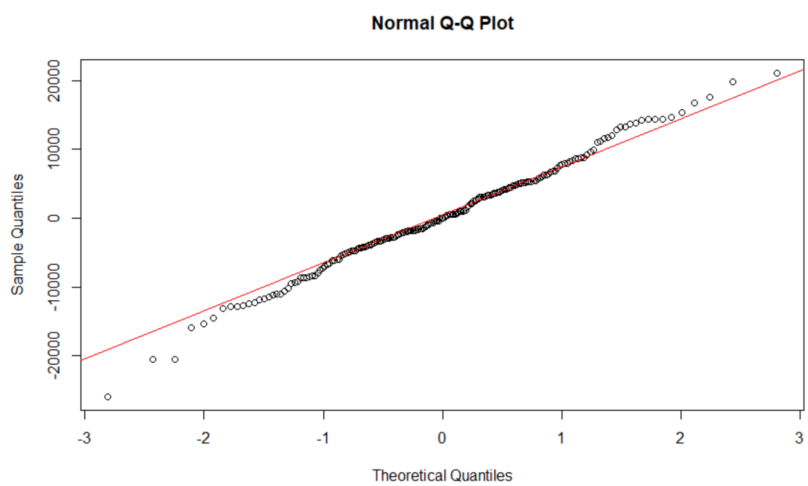


Figure 6.4: VaR estimation of Portfolio 1. The 95% and 99% VaR are the tenth and second worst loss, respectively.

```
> #We estimate the 1-day 95% VaR, which is the tenth worst loss
> VaR95 = scenario[10,4]
> VaR95
[1] 14204.79
>
> #We estimate the 1-day 99% VaR, which is the second worst loss
> VaR99 = scenario[2,4]
> VaR99
[1] 19781.03
```

6.2.2 Portfolio 2

A summary of results, in figure 6.5, shows that the greatest loss that can occur is 17,215 (under Scenario 28). The histogram of returns in figure 6.6 and the normal Q-Q plot in figure 6.7 indicate that we may assume that the daily returns follow the normal distribution.

The VaR of Portfolio 2, in figure 6.8, is estimated:

- With 95% confidence we can say that the maximum 1-Day loss will be 6,178.94 €.
- With 99% confidence we can say the maximum 1-Day loss will be 9,111.05 €.

Figure 6.5: Summary of the returns and Portfolio 2 values

```
   nscenario        EURO.USD        portfolio          returns
 Min.   :  1.0   Min.   :1.138   Min.   : 982785   Min.   :-13798.8
 1st Qu.:117.8   1st Qu.:1.156   1st Qu.: 998255   1st Qu.: -2539.5
 Median :234.5   Median :1.158   Median :1000000   Median :     0.0
 Mean   :234.5   Mean   :1.158   Mean   :1000126   Mean   :  -126.1
 3rd Qu.:351.2   3rd Qu.:1.161   3rd Qu.:1002540   3rd Qu.:  1744.9
 Max.   :468.0   Max.   :1.174   Max.   :1013799   Max.   : 17215.0
```

Figure 6.6: Histogram of returns (Portfolio 2)

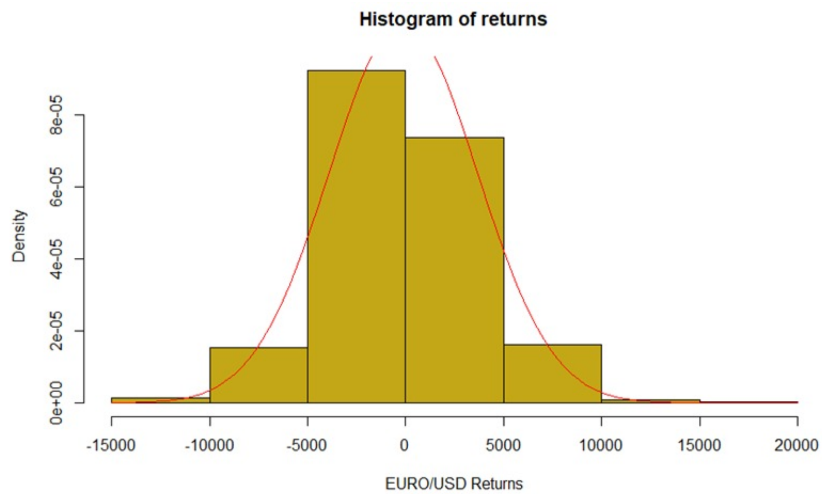


Figure 6.7: Q-Q plot of the EURO/SD exchange rates from Portfolio 2

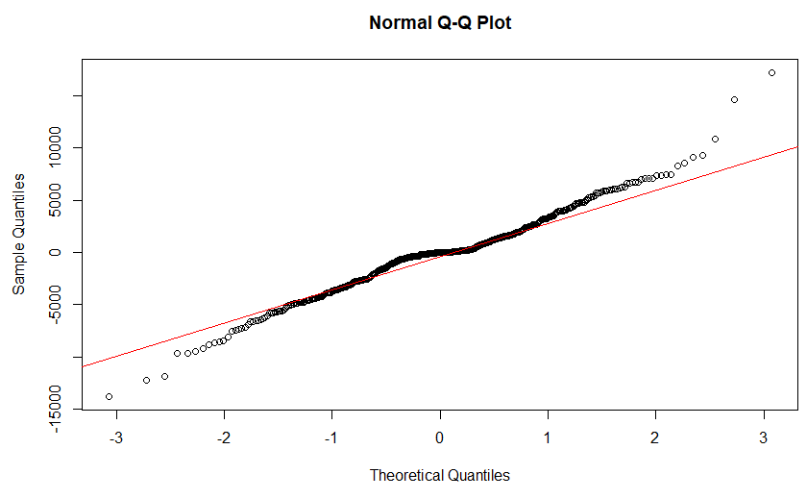


Figure 6.8: VaR estimation of Portfolio 2. The 95% and 99% VaR are the 23rd and 5th loss, respectively

```
> #we estimate the 1-day 95% VaR, which is the 23rd worst loss
> VaR95 = scenario[23,4]
> VaR95
[1] 6178.942
>
> #we estimate the 1-day 95% VaR, which is the 5th worst loss
> VaR99 = scenario[5,4]
> VaR99
[1] 9111.056
```

6.2.3 Portfolio 3

At last we give a interpretation of the results we obtained in regards to Portfolio 3.

The summary of the returns is given in figure 6.9. It shows that the worst loss is SRD 127,572.2 (under Scenario 55) and the maximum portfolio value is 1,434,090.

With the histogram of losses, in figure 6.10, we can already identify that the returns do not follow the normal distribution since it shows a long tail extending to the left. There is no symmetry and great amount of skewness can be detected. Essentially, there would have to be zero skewness to conclude a normal distribution. Furthermore, the Q-Q plot (figure 6.3) indicates that distribution of the returns is far from normal.
The VaR of Portfolio 3, seen in figure 6.11, is estimated:

- With 95% confidence we can say that the maximum 1-Day loss will be SRD 13,615.42.
- With 99% confidence we can say the maximum 1-Day loss will be SRD 121,218.50.

Figure 6.9: Summary of the returns and Portfolio 3 values

```
  nscenario        EURO.SRD        portfolio          returns
 Min.   :  1.0   Min.   :21.18   Min.   : 872428   Min.   :-434090.4
 1st Qu.: 39.5   1st Qu.:24.21   1st Qu.: 997311   1st Qu.:  -3443.8
 Median : 78.0   Median :24.27   Median : 999941   Median :     59.2
 Mean   : 78.0   Mean   :24.35   Mean   :1002950   Mean   :  -2950.2
 3rd Qu.:116.5   3rd Qu.:24.36   3rd Qu.:1003444   3rd Qu.:   2689.3
 Max.   :155.0   Max.   :34.81   Max.   :1434090   Max.   : 127572.2
```

Figure 6.10: Histogram of the returns (Portfolio 3)

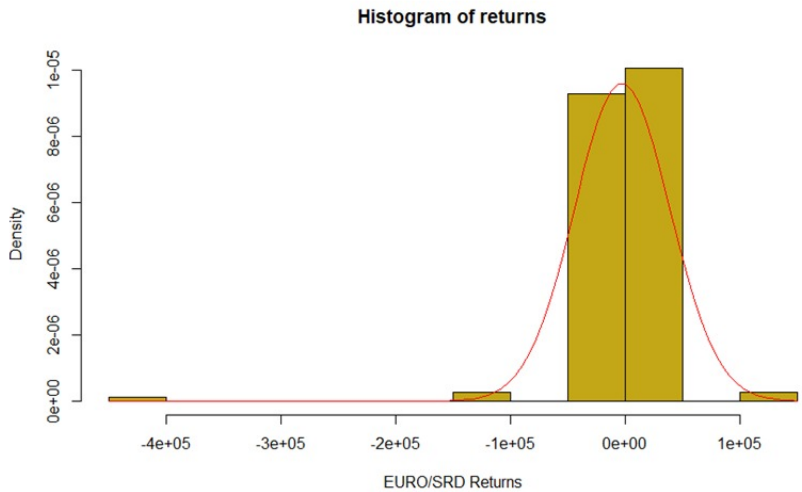


Figure 6.11: Q-Q plot of the EURO/SRD exchange rates returns from Portfolio 3

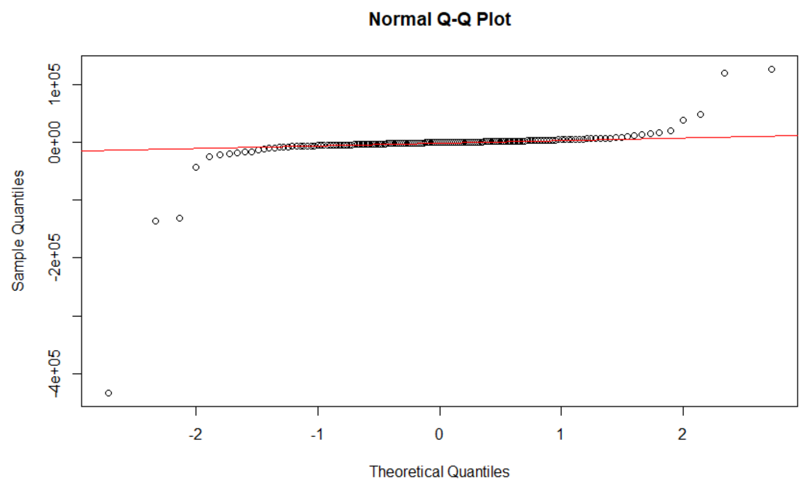


Figure 6.12: The 95% and 99% VaR are estimated, which are the eight and second worst loss respectively

```
> #We estimate the 1-day 95% VaR, which is the 8th worst loss
> VaR95 = scenario[8,4]
> VaR95
[1] 13615.42
>
> #We estimate the 1-day 95% VaR, which is the 2nd worst loss
> VaR99 = scenario[2,4]
> VaR99
[1] 121218.5
```

Chapter 7

Conclusion

In this first part of this study we gained a better understanding of the basics of financial markets, risk modeling and valuation with models such as binomial trees and Black-Scholes model.

In the last part of this study we focused on estimating the market risk for EURO/SRD and EURO/USD exchange rates using the historical simulation method of Value at Risk. We used this approach since it does not assume the returns to follow a normal distribution. Another advantage of this method is that the estimation of the risk only depends on the returns. However, it needs all the risk factors that are obtainable over a certain historic time. Further, we have seen that the losses in our EURO/SRD exchange rates is driven by large spikes which occur in combination with sudden declines of the FX rates on specific dates followed by a pretty stable period (figure 2.6). VaR measure which is based on a specific period may therefore miss this risk. Hence, it is not recommended to use this VaR for estimation for the market risk. For further study, we recommend to obtain more historical data of the EURO/SRD exchange rates and to perform stress tests of VaR. Stress testing will provide a better estimation of the market risk for these foreign exchange rates. It would also be interesting to compare the results of the stress tests to the historical VaR estimation.

Appendices

Appendix A

Derivation of the Black-Scholes-Merton equation

The Black-Scholes-Merton equation is a partial differential equation developed by three economists Fischer Black, Myron Scholes and Robert Merton. This equation is used to price European call and put options on an underlying stock with no dividends. In this Appendix I derive the equation that deduces the model in Section 4.3.1.

When it comes to deriving the Black-Scholes equation we first need to assume that:

1. The stock price follows a stochastic process.
2. There are no dividend and transaction costs.
3. There are no arbitrage opportunities.
4. The risk-free rate, r, is known and constant.

As we've already looked at the Geometric Brownian motion and Itô's lemma, there is one more component we need to understand before deriving this equation. This is called the **Delta-hedge portfolio** used to hedge the risk on an underlying stock.

First, let's suppose that P is the price of a call. So P is a function of S and t.

we have:

$$dP = (\frac{\partial P}{\partial t} + \frac{\partial P}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 P}{\partial^2 S}\sigma^2 S^2)dt + \frac{\partial P}{\partial S}\sigma S dz \tag{A.1}$$

The Delta-hedge portfolio consists of a call with value -P and $\frac{\partial P}{\partial S}$ shares:

$$\Pi = \frac{\partial P}{\partial S}S - P \tag{A.2}$$

The change $d\Pi$ in value of the portfolio is then:

$$d\Pi = \frac{\partial P}{\partial S}dS - dP \tag{A.3}$$

Substituting equation (4.8) in (4.10) we get:

$$d\Pi = \frac{\partial P}{\partial S}dS - (\frac{\partial P}{\partial t} + \frac{\partial P}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 P}{\partial^2 S}\sigma^2 S^2)dt - \frac{\partial P}{\partial S}\sigma S dz \tag{A.4}$$

In section 3.3 we saw that $dS = \mu S dt + \sigma S dz$
So, (A.4) becomes

$$d\Pi = -\frac{\partial P}{\partial t} - \frac{1}{2}\frac{\partial^2 P}{\partial^2 S}\sigma^2 S^2 dt \tag{A.5}$$

Since P doesn't depend on dz, this is a riskless portfolio. Since the value of the portfolio is also independent on μ, the expected rate of return, then it must increase at the same rate of return, i.e.,

$$d\Pi = r\Pi dt \tag{A.6}$$

By substituting (A.2) and (A.5) into (A.6) we get

$$\frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial^2 P}{\partial^2 S}\sigma^2 S^2 dt = r(P - \frac{\partial P}{\partial S}S) \tag{A.7}$$

And now we obtain the Black-Scholes-Merton equation,

$$\frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial^2 P}{\partial^2 S}\sigma^2 S^2 dt + rs\frac{\partial P}{\partial S} - rP = 0 \tag{A.8}$$

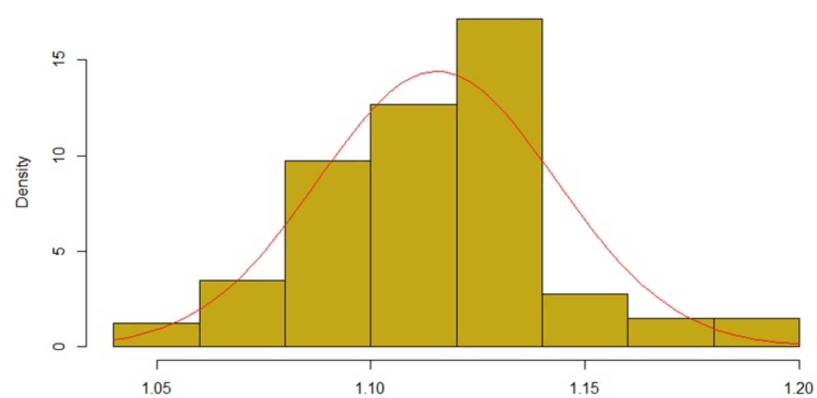
In the Black-Scholes-Merton equation there is risk neutral valuation, meaning that the price is not defined by the risk preferences of market shareholders. This is because it doesn't involve μ, which does depend on risk preferences. The value of a forward contract also holds the risk neutral valuation property.

Appendix B

Distributions of the data

In this appendix we have the distributions of the foreign exchange rates data given in histograms. The graph, in figure B.1, indicates to be close to a bell

Figure B.1: Distribution of the EURO/USD exchange rates from January 2, 2015, to October 15, 2015. The base FX rate here is 1.1386.



curve, further indicating that the distribution of the data is close to normal. In figure B.2, the data seems to be less symmetrical compared to the one mentioned before. The distribution of the EURO/SRD exchange rates, in figure B.3 looks extremely skewed, suggesting that it is far from normal.

Additionally, the 95% and 99% quantile for each portfolio, corresponds with the 5% and 1% largest loss from the 200, 468 and 155 scenarios.

Figure B.2: Distribution of the EURO/USD exchange rates from April 1, 2020, to September 30, 2021. The base FX rate here is 1.1581.

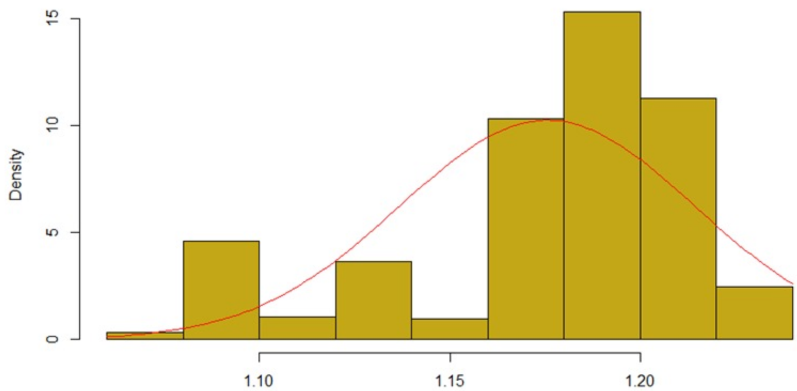
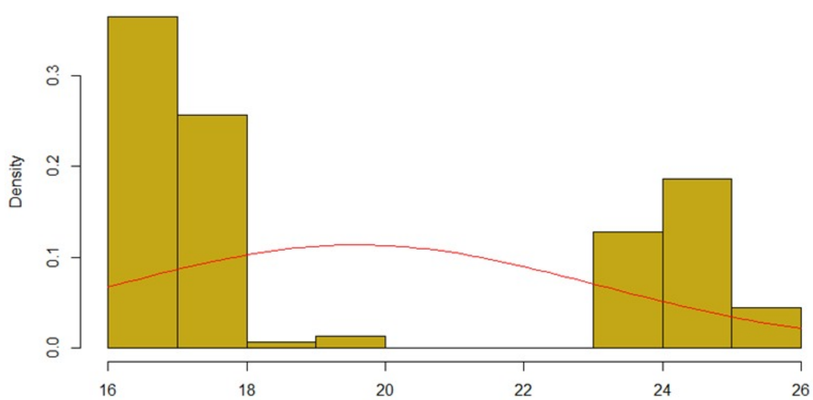


Figure B.3: Distribution of the EURO/SRD exchange rates from January 4, 2021, to September 1, 2021. The base FX rate here is 24.275.



Appendix C

R scripts

```r
#Historical simulation for EURO/SRD exchange rates from 1/4/2021 - 9/1/2021

#After the data is imported, we view the first 5 rows
data= EURO_SRD_exchange_rates
data[1:5,]

#All the scenarios for our portfolio are being created (with observations from the past)
n=nrow(data)
scenario=array(c(0),c(n-1,1))
for (i in 2:n){scenario[i-1]=(data[i,3]/data[i-1,3]*data[n,3])}
colnames(scenario)= c('EURO/SRD')
scenario[1:10]

#Counter for the scenarios
nscenario=c(1:155)

#Now we calculate the value of our portfolio tomorrow
portfolio = c()
for (i in 1:n-1){portfolio[i]= 1000000*scenario[i]/data[n,3]}
portfolio[1:10]

#The losses/returns are calculated
returns=1000000-portfolio
returns[1:20]

#Combine everything in one data frame
scenario=cbind(nscenario,scenario,portfolio,returns)
```

```
scenario=data.frame(scenario)
scenario[1:5,]

#A summary of our data set
summary(scenario)

#The distribution of our losses/returns in a histogram and normal Q-Q plot
hist(returns,xlab = "EURO/SRD Returns" ,freq = FALSE, col="#C4A716")
curve(dnorm(x,mean=mean(returns),sd=sd(returns)), add=TRUE, col="red")
summary(returns)
qqnorm(returns)
qqline(returns, col="red")

#We perform a Shapiro-Wilk normality test
shapiro.test(returns)

#We sort the losses/returns from the worst to the best
scenario=scenario[sort.list(scenario[,4],decreasing=TRUE), ]
scenario[1:5,]

#We estimate the 1-day 95% VaR, which is the 8th worst loss
VaR95 = scenario[8,4]
VaR95

#We estimate the 1-day 95% VaR, which is the 2nd worst loss
VaR99 = scenario[2,4]
VaR99
```

```
#Historical simulation for EURO/USD exchange rates from 1/2/2015 - 10/15/2015

#After the data is imported, we view the first 5 rows
data= euro_dollar_exchange_rates
data[1:5,]

#All the scenarios for our portfolio are being created (with observations from the past)
n=nrow(data)
scenario=array(c(0),c(n-1,1))
for (i in 2:n){scenario[i-1]=(data[i,3]/data[i-1,3]*data[n,3])}
colnames(scenario)= c('EURO/USD')
scenario[1:10]
```

```

#Counter for the scenarios
nscenario=c(1:200)

#Now we calculate the value of our portfolio tomorrow
portfolio = c()
for (i in 1:n-1){portfolio[i]= 1000000*scenario[i]/data[n,3]}
portfolio[1:10]

#The losses/returns are calculated
returns=1000000-portfolio
returns[1:20]

#Combine everything in one data frame
scenario=cbind(nscenario,scenario,portfolio,returns)
scenario=data.frame(scenario)
scenario[1:5,]

#A summary of our data set
summary(scenario)

#The distribution of our losses/returns in a histogram and normal Q-Q plot
hist(returns,xlab = "EURO/USD Returns" ,freq = FALSE,col= "#C4A716")
curve(dnorm(x,mean=mean(returns),sd=sd(returns)), add=TRUE, col="red")
summary(returns)
qqnorm(returns)
qqline(returns, col="red")

#We sort the losses/returns from the worst to the best
scenario=scenario[sort.list(scenario[,4],decreasing=TRUE), ]
scenario[1:5,]

#We estimate the 1-day 95% VaR, which is the tenth worst loss
VaR95 = scenario[10,4]
VaR95

#We estimate the 1-day 99% VaR, which is the second worst loss
VaR99 = scenario[2,4]
VaR99
```

```
#Historical simulation for EURO/USD exchange rates from 4/1/2020 - 9/30/2021
```

```

#After the data is imported, we view the first 5 rows
data= euro_dollar_exchange_rates_2020.2021
data[1:5,]

#All the scenarios for our portfolio are being created (with observations from the past)
n=nrow(data)
scenario=array(c(0),c(n-1,1))
for (i in 2:n){scenario[i-1]=(data[i,3]/data[i-1,3]*data[n,3])}
colnames(scenario)= c('EURO/USD')
scenario[1:5]

#Counter for the scenarios
nscenario=c(1:468)

#Now we calculate the value of our portfolio tomorrow
portfolio = c()
for (i in 1:n-1){portfolio[i]= 1000000*scenario[i]/data[n,3]}
portfolio[1:5]


#The losses/returns are calculated
returns=1000000-portfolio
returns[1:5]

#Combine everything in one data frame
scenario=cbind(nscenario,scenario,portfolio,returns)
scenario=data.frame(scenario)
scenario[1:5,]

#A summary of our data set
summary(scenario)

#The distribution of our losses/returns in a histogram and normal Q-Q plot
hist(returns,xlab = "EURO/USD Returns" ,freq = FALSE, col = "#C4A716")
curve(dnorm(x,mean=mean(returns),sd=sd(returns)), add=TRUE, col="red")
summary(returns)
qqnorm(returns)
qqline(returns, col="red")

```

```
#We sort the losses/returns from the worst to the best
scenario=scenario[sort.list(scenario[,4],decreasing=TRUE), ]
scena rio[1:5,]

#We estimate the 1-day 95% VaR, which is the 23rd worst loss
VaR95 = scenario[23,4]
VaR95

#We estimate the 1-day 95% VaR, which is the 5th worst loss
VaR99 = scenario[5,4]
VaR99
```

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